Modeling and Analysis of Human Behavior based on Hybrid System Model

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Abstract. This paper presents a development of the modeling strategy of the human driving behavior based on the Hybrid System model focusing on the driver’s collision avoidance maneuver. In our modeling, the control scenario of the human driver, that is, the mapping from the driver’s sensory information to the operation of the driver such as acceleration, braking and steering, is expressed by Piecewise ARX model. This paper addresses the cases that the switching between sub-ARX models is caused in both deterministic logic-based and stochastic manners. Then, the identification problems for the logic-based/stochastic switched ARX models (LS-ARX and SS-ARX models) are formulated and solved. The LS-ARX model enables us to capture not only the physical meaning of the driving skill, but also the decision-making aspect (switching conditions) in the driver’s collision avoidance maneuver. Also, the SS-ARX model can be used for the behavior recognition which can be regarded as natural extension of the speech recognition.

1 Introduction

Recently, the modeling of driving behavior has attracted much attention by many researchers[1][2][4][9]. In order to model the driver’s driving maneuver, the conventional techniques such as the nonlinear regression models, the neural network and fuzzy systems have been used [17]. These techniques, however, have some problems as follows: (1) the obtained model often results in too complicated model, (2) this makes it impossible to understand the physical meaning of the driver’s maneuver. When we look at the driving behavior, it is often found that the driver appropriately switches the simple control laws instead of adopting the complex nonlinear control law. The switching mechanism can be regarded as a kind of driver’s decision-making in the driver’s driving behavior. Therefore, it is highly recommended that the model of the driving behavior involves both physical skill (motion) and the decision-making aspect (switching condition). This kind of expression can be categorized into a class of Hybrid Dynamical Systems (HDSs). HDSs are systems, which consist of both continuous dynamics and logical conditions. The former are typically associated with the differential (or difference) equations, the latter with combinatorial logics, automata and so
Although many literatures have dealt with the description, stability analysis, control verification and parameter identification of the HDS, the application to the human behavior analysis has not been discussed as far as the authors know. In this paper, the Piecewise ARX model, which is a class of the HDS, is adopted to understand the human driving behavior especially focusing on the driver’s collision avoidance maneuver. In our modeling, the relationship among the observed information such as the sensory information of range between cars, range rate, lateral displacement between cars and the steering output of driver is expressed by the Piecewise ARX model. This paper addresses the cases that the switching between sub-ARX models is caused in both deterministic logic-based and stochastic manners. Then, the identification problems for the logic-based/stochastic switched ARX models (hereafter, called LS-ARX and SS-ARX models) are formulated and solved.

The LS-ARX model enables us to capture not only the physical meaning of the driving skill, but also the decision-making aspect (switching conditions) in the driver’s collision avoidance maneuver. Although there are several techniques to identify the parameters of LS-ARX model [8][9][10][11], the clustering based approach is introduced in this paper, which is originally developed in [10], since it is applicable to large data set. In this case, the observed data set is categorized into some modes, then the logical switching condition between modes are found by applying Support Vector Machine (SVM) technique.

On the other hand, the SS-ARX model can be regarded as a natural extension of the Hidden Markov Model (HMM) which shows great performance in the field of speech recognition[12]. This was partly because that the speech signal inherently has a ‘piecewise’ characteristics in it. The ideas developed for the speech recognition are also applicable to gesture recognition, which is another way of communication. Furthermore, a biometrics based on the behavioral information such as walking, grasping, and so on, is now considered as promising application area. The motivation of this comes from the fact that complex behavior consists of primitive dynamics, which is often referred as motion primitives [14], movemes [13], and so on. However, in these works, the explicit combination of the continuous dynamics and the stochastic discrete model has not been fully discussed as far as the authors know. In [15], the parameter estimation problem for the stochastic hybrid system was addressed where autonomous state equation is assigned to each discrete state as a generator of the observed signal at each discrete state. However, the input-output relation of the system was not considered explicitly. This may cause inconvenience when this model is applied to behavior recognition. On the other hand, the ARX model directly represents dynamical relationship between input and output signals. Therefore, the SS-ARX model is expected to play an essential role in the behavior recognition. The parameter estimation algorithm for the SS-ARX model is derived based on the EM algorithm as well as the one for the HMM.

In this paper, we also verify the potential performance of the LS-ARX and SS-ARX models as the basic system model to analyze the human behavior through the experiments on the drivers collision avoidance maneuver. The driving data
are collected by using the three-dimensional driving simulator based on CAVE, which provides stereoscopic immersive vision. Thanks to the stereoscopic vision, the examinee can feel almost real driving feelings in our experiments.

2 Parameter estimation of LS-ARX model

In this section, we introduce the parameter estimation algorithm for the hybrid system which consists of sub-ARX models together with the logic-based switching conditions between them (See [10] in detail, hereafter, we call this Logic-based Switched ARX model: LS-ARX model). The mathematical form in the case of the number of modes is three is given as follows:

\[
y_t = \begin{cases} 
  \psi_t^T \theta_0 + e_t, & \text{if } \psi_t \in C_0 \\
  \psi_t^T \theta_1 + e_t, & \text{if } \psi_t \in C_1 \\
  \psi_t^T \theta_2 + e_t, & \text{if } \psi_t \in C_2
\end{cases}
\]  

(1)

where \(y_t\) and \(\psi_t\) are an output and regressor vector of the system at \(t\). \(e_t\) is called an equation error, and is supposed to have a Gaussian distribution with variance \(\sigma\).

2.1 Mode classification by data clustering

In order to identify the parameters in the LS-ARX model, first of all, the input and output data are classified, and categorized into the corresponding modes. The outline of the clustering is described as follows:

- Suppose that the sample data series \(\{o_t\} = \{(y_t, \psi_t)\} (t = 0, 1, \ldots, T)\) is given. For each sample data \(o_t\) \((t = 0, 1, \ldots, T)\), collect the neighboring \(c\) data, generate the data set \(LDs_t\), and find the feature vector \(\xi_t\) (Fig.1(a)). Here, the feature vector \(\xi_t\) consists of the parameters \(\theta\) calculated by least mean square method and the mean value of the data among the \(LDs_t\).
- Transform the series of sample data \(\{o_t\}\) to the series of feature vectors \(\{\xi_i\} (i = 1, 2, \ldots, n)\) (Fig.1(b)). Apply the K-means algorithm [16] to the \(\{\xi_i\}\) (Fig.1(c)).
- Transform the clustering results in the feature vector space to one in the original sample data space (Fig.1(d)).

The parameters of each ARX model are found by applying the standard least mean square method to the sample data set in each mode defined by the clustering procedure.

2.2 Identification of switching plane by SVM

The next step is to find the switching plane from one cluster to other, that is, the boundaries between regions \((C_0, C_1 \text{ and } C_2 \text{ in (1)})\). As a useful tool for this,
the Support Vector Machine (SVM) is often used, where the plane is restricted to the linear hyperplane. The outline of SVM is briefly reviewed as follows:

SVM is a kind of learning procedure which classifies data into two classes by finding linear hyperplane. Suppose that the training data $x_i$ and Label for class $t_i \in \{-1, 1\}$ are given. If the training data are distinguishable by the linear hyperplane, the following parameters $\{w, h\}$ exist.

$$w^T x_i - h \geq 1 \text{ if } t_i = 1$$
$$w^T x_i - h \leq -1 \text{ if } t_i = -1 \quad (2)$$

SVM tries to find these parameters by maximizing the distance between two hyperplanes $w^T x_i - h = 1$ and $w^T x_i - h = -1$, which is given by $\frac{1}{\|w\|}$ (Maximization of margin, see Fig.2). This problem can be formulated as the following quadratic programming.

$$\min L(w) = \frac{1}{2} \|w\|^2$$
$$\text{subject to } t_i(w^T x_i - h) \geq 1, \quad i = 1, \ldots, N \quad (3)$$

If the training data are not distinguishable, the original quadratic programming is reformulated by introducing the nonnegative variable $\eta_i$ as follows:
\[ \min L(w) = \frac{1}{2} \|w\|^2 + \mu \sum_{i=1}^{N} \eta_i \]
\[ \text{s.t. } t_i(w^T x_i - h) \geq 1 - \eta_i, \]
\[ \eta_i \geq 0, \quad i = 1, \ldots, N \] (4)

The variable \( \eta_i \) represents the penalty for exceeding the hyperplane, and \( \mu \) denotes the weight for the penalty. This relaxation of the constraints is called a Soft Margin technique. In this work, we use this relaxed formulation to find the switching planes.

### 3 Parameter estimation of SS-ARX model

In this section, we consider the parameter estimation problem of the stochastic hybrid system which consists of sub-ARX models together with the probabilistic switching between them (Hereafter, we call this Stochastic Switched ARX model: SS-ARX model). The three state SS-ARX model is depicted in Fig.3

#### 3.1 Parameters in SS-ARX model

The parameters in SS-ARX model are specified as follows:

- Set of discrete states \( S = \{S_i, (i=0, 1, \ldots, N)\} \)
- \( a_{ij} \): State transition probability \( (i=0, 1, \ldots, N; j=0, 1, \ldots, N) \)
- \( \pi_i \): Initial state probability \( (i=0, 1, \ldots, N) \)
- \( \theta_i \): Parameters in ARX model assigned to \( S_i \) \( (i=0, 1, 2, \ldots, N) \)
- \( \sigma_i \): Variances of equation error in ARX model assigned to \( S_i \) \( (i=0, 1, 2, \ldots, N) \)

\( N + 1 \) denotes the number of discrete states.
3.2 Definition of observed symbol in SS-ARX model

In order to derive the parameter estimation algorithm, a observed symbol and its occurrence probability are defined for the SS-ARX model as follows: By this definition, the parameter estimation algorithm for HMM can be naturally extended to the one for SS-ARX model. First of all, the observed symbol \( o_t \) at time \( t \) is defined as the combination of the output \( y_t \) and the regressor \( \psi_t \), that is, \( o_t = (y_t, \psi_t) \). Then, its occurrence probability \( b_i(o_t) \) is defined by the assumption of the Gaussian distribution of the equation error.

\[
b_i(o_t) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp \left\{ -\frac{(\psi_t^T \theta_i - y_t)^2}{2\sigma_i^2} \right\} . \tag{5}
\]

By these definitions, the parameter estimation problem for SS-ARX model can be regarded as natural extension of the problem for HMM with continuous observed symbol.

3.3 Parameter Estimation

**EM algorithm** For the ARX model, in which its parameters are denoted by \( \lambda \), we consider an unobservable state sequence \( s = (s_0, s_1, \ldots, s_t, \ldots, s_T) \) and an observable symbol sequence \( O = (o_0, o_1, \ldots, o_t, \ldots, o_T) \). The maximization of the likelihood value of the \( s \) and \( O \), \( L(s, O; \lambda) = P(s, O|\lambda) \) is achieved by introducing the EM algorithm. EM algorithm can locally maximize the likelihood value \( L(s, O; \lambda) = P(s, O|\lambda) \) by iterative procedure.

Generally, the EM algorithm tries to find the parameter \( \lambda' \) which maximizes the following Q function:

\[
Q(\lambda, \lambda') = E[\log \{P(s, O|\lambda')\}] | O, \lambda
\]

\[
= \sum_s P(s|O, \lambda) \log \{P(s, O|\lambda')\} \tag{7}
\]
by executing following procedures iteratively.

1. Specify an initial parameter $\lambda$.
2. Find the $\lambda'$ which maximizes the $Q(\lambda, \lambda')$.
3. Substitute $\lambda'$ for $\lambda$, and iterate 2) until $\lambda' = \lambda$ holds.

**Parameter estimation algorithm** The parameters of SS-ARX model before and after the update are supposed to be given by $\lambda = (\pi_i, a_{ij}, \theta_i, \sigma_i)$ and $\lambda' = (\pi'_i, a'_{ij}, \theta'_i, \sigma'_i)$. From (7),

$$Q(\lambda, \lambda') = \sum_s P(s|O, \lambda) \log P(s, O|\lambda') \quad (8)$$

$$= \frac{1}{P(O|\lambda)} \sum_s P(s, O|\lambda) \log P(s, O|\lambda'). \quad (9)$$

Since $\frac{1}{P(O|\lambda)}$ is constant, the maximization of $Q(\lambda, \lambda')$ implies the maximization of $\tilde{Q}(\lambda, \lambda')$ given by

$$\tilde{Q}(\lambda, \lambda') = \sum_s P(s, O|\lambda) \log P(s, O|\lambda'). \quad (10)$$

By using the definition

$$P(s, O|\lambda) = \pi_{s_0} b_{s_0} (o_0) \times a_{s_0 s_1} b_{s_1} (o_1) \times \cdots \times a_{s_{T-1} s_T} b_{s_T} (o_T), \quad (11)$$

the $\tilde{Q}(\lambda, \lambda')$ can be decomposed as follows:

$$\tilde{Q}(\lambda, \lambda') = \tilde{Q}_1(\lambda, \lambda') + \tilde{Q}_2(\lambda, \lambda') + \tilde{Q}_3(\lambda, \lambda') \quad (12)$$

where

$$\tilde{Q}_1(\lambda, \lambda') = \sum_{s_0=0}^{N} \sum_{s_1=0}^{s_0} \cdots \sum_{s_T=0}^{s_0} \pi_{s_0} b_{s_0} (o_0) \times a_{s_0 s_1} b_{s_1} (o_1) \times \cdots \times a_{s_{T-1} s_T} b_{s_T} (o_T) \times \log \{\pi'_{s_0}\} \quad , (13)$$

$$\tilde{Q}_2(\lambda, \lambda') = \sum_{s_0=0}^{N} \sum_{s_1=0}^{s_0} \cdots \sum_{s_T=0}^{s_0} \pi_{s_0} b_{s_0} (o_0) \times a_{s_0 s_1} b_{s_1} (o_1) \times \cdots \times a_{s_{T-1} s_T} b_{s_T} (o_T) \times \sum_{t=1}^{T} \log \{a'_{s_{t-1} s_t}\} \quad , (14)$$

$$\tilde{Q}_3(\lambda, \lambda') = \sum_{s_0=0}^{N} \sum_{s_1=0}^{s_0} \cdots \sum_{s_T=0}^{s_0} \pi_{s_0} b_{s_0} (o_0) \times a_{s_0 s_1} b_{s_1} (o_1) \times \cdots \times a_{s_{T-1} s_T} b_{s_T} (o_T) \times \sum_{t=0}^{T} \log \{b'_{s_t} (o_t)\} \quad (15)$$
Next, the forward probability $\alpha(i, t)$ and backward probability $\beta(i, t)$ are defined as follows:

$$
\alpha(i, t) = \sum_{s_0=0}^{N} \sum_{s_1=0}^{N} \cdots \sum_{s_{t-1}=0}^{N} \pi_{s_0} b_{s_0}(o_0) \times a_{s_0s_1} b_{s_1}(o_1) \times \cdots \times a_{s_{t-1}s_t} b_{s_t}(o_t) \quad (16)
$$

$$
\beta(i, t) = \sum_{s_{t+1}=0}^{N} \sum_{s_{t+2}=0}^{N} \cdots \sum_{s_T=0}^{N} a_{s_{t+1}s_{t+2}} b_{s_{t+2}}(o_{t+2}) \times \cdots \times a_{s_{T-1}s_T} b_{s_T}(o_T) \quad (17)
$$

The meaning of $\alpha(i, t)$ is the probability for the SS-ARX model $\lambda$ to generate the symbol sequence $O = (o_0, o_1, \cdots, o_t)$ until $t$ and reach the state $S_t$ at $t$ (i.e. $s_t = S_t$). Also, the meaning of $\beta(i, t)$ is the probability for the SS-ARX model $\lambda$ to generate the symbol sequence $O = (o_{t+1}, o_{t+2}, \cdots, o_T)$ starting from $S_t$ at $t$ (i.e. $s_t = S_t$) and reach the final state at $T$.

By using eqs. (16) and (17), eqs. (13), (14) and (15) are rewritten as follows:

$$
\tilde{Q}_1(\lambda, \lambda') = \sum_{i=0}^{N} \pi_i \log \{ \pi_i' \} b_i(o_0) \beta(i, 0) \quad (18)
$$

$$
\tilde{Q}_2(\lambda, \lambda') = \sum_{i=0}^{T} \sum_{j=0}^{N} \log \{ a_{ij} \} \alpha(i, t - 1) a_{ij} b_i(o_t) \beta(j, t) \quad (19)
$$

$$
\tilde{Q}_3(\lambda, \lambda') = \sum_{i=0}^{T} \sum_{t=0}^{N} \log \{ b_i'(o_t) \} \alpha(i, t) \beta(i, t) \quad (20)
$$

By maximizing $\tilde{Q}_1(\lambda, \lambda')$, $\tilde{Q}_2(\lambda, \lambda')$ and $\tilde{Q}_3(\lambda, \lambda')$, $\tilde{Q}(\lambda, \lambda')$ can be maximized. Therefore, $\lambda'$ which maximizes the $\tilde{Q}(\lambda, \lambda')$ can be obtained as follows:

$$
\pi_i' = \frac{\pi_i b_i(o_0) \beta(i, 0)}{\sum_{k=0}^{N} \pi_k b_k(o_0) \beta(k, 0)} \quad (21)
$$

$$
a_{ij}' = \frac{\sum_{t=0}^{T} \alpha(i, t - 1) a_{ij} b_j(o_t) \beta(j, t)}{\sum_{k=0}^{N} \sum_{t=1}^{T} \alpha(i, t - 1) a_{ik} b_k(o_t) \beta(k, t)} \quad (22)
$$

$$
\theta_i' = \left\{ \sum_{t=0}^{T} \left\{ \psi_t \psi_t^T \alpha(i, t) \beta(i, t) \right\} \right\}^{-1} \left\{ \sum_{t=0}^{T} \left\{ \psi_t y_i \alpha(i, t) \beta(i, t) \right\} \right\} \quad (23)
$$

$$
\sigma_i'^2 = \frac{\sum_{t=0}^{T} \left\{ \psi_t^T \theta_i' - y_i \right\}^2 \alpha(i, t) \beta(i, t)}{\sum_{t=0}^{T} \left\{ \alpha(i, t) \beta(i, t) \right\}} \quad (24)
$$

By iterating three steps in the EM algorithm together with (21), (22), (23) and (24), the parameter $\lambda$ is locally optimized. Note that eq. (23) can be regarded as the weighted least mean square solution in which the weight parameters are specified by the sojourn probability at the state $S_i$. 
4 Application to drivers behavior modeling

4.1 Configuration of Driving Simulator

The configuration and appearance of the developed DS based on the CAVE are shown in Figs. 4(a) and (b). The display unit in the CAVE system provides the 3D virtual environment, and it is controlled by ONYX2. The display program was developed by making use of the CAVE library and the Performer. The cockpit is built by installing a handle, an accelerator and a brake in the CAVE system. The information on the driver’s output to the handle, accelerator and brake is transferred to the PC through the USB terminal, and the vehicle position and motion are calculated based on these inputs and vehicle dynamics implemented on the PC using the CarSim software. The results of the calculation are transferred to ONYX2 through the Internet (TCP/IP), and the 3D visual image based on the position and motion of the vehicle is displayed.

4.2 Data acquisition in collision avoidance maneuver

**Experimental environment and conditions** In this paper, we focus on the driver’s collision avoidance behavior at the instance of the sudden stopping of the preceding vehicle when the examinee looks aside from the road ahead. In order to model the driver’s collision avoidance maneuver, the following sensory information is captured as the inputs:

1. Range between cars \((x_{1,t})\)
2. Range rate \((x_{2,t})\)
3. Lateral displacement between cars \((x_{3,t})\)

The outputs of drivers are also specified as follows:

1. Steering amount \((y_t)\)

The selection of sensory information was motivated by the result in [9]. One may argue that the braking operation should be taken into account the model. However, thanks to the enough range between cars, the influence of the braking is smaller than that of steering.

In our modeling, the regressor vector was specified as follows:

\[
\psi_t = (x_{1,t-m}, x_{2,t-m}, x_{3,t-m})^T
\]  

(25)

where \(m\) represents the time delay from the perception to the operation in human behavior. In the analyses shown in the later sections, \(m\) was set to be twelve. Since the sampling interval was 16[msec], this implies that we have specified about 200[msec] time delay, which is known as a reasonable value in the field of human engineering. Also, the number of discrete states (modes) in both the LS-ARX and SS-ARX models was set to be three. Note that although the autoregression terms, i.e. the past values of the output \(y_t\) are not included in the regressor vector, extension to such case is straightforward.
Fig. 4. The developed driving simulator based on CAVE.
**Procedure of experiment** The examinee drives the car with keeping constant distance to the preceding truck. The operator set the red or green parking vehicles on the right side at the intersection. The examinee are supposed to take a look at the right side at each intersection, then the examinee answers what colors the parking vehicle is and/or whether there are any parking vehicles. When the examinee looks right side at some intersection, the preceding truck is supposed to stop suddenly with maximum deceleration. Then, the collision avoidance behavior of the examinee is observed.

4.3 Identification as LS-ARX model

In this subsection, the LS-ARX model is introduced as a mathematical model of the driving behavior. We consider the following LS-ARX model which consists of three modes:

*mode A (Avoidance)*

\[ y_t = \theta_{0,1}x_{1,t-m} + \theta_{0,2}x_{2,t-m} + \theta_{0,3}x_{3,t-m} \quad \text{if} \quad \psi_t \in C_0 \]  

(26)

*mode B (Overtake)*

\[ y_t = \theta_{1,1}x_{1,t-m} + \theta_{1,2}x_{2,t-m} + \theta_{1,3}x_{3,t-m} \quad \text{if} \quad \psi_t \in C_1 \]  

(27)

*mode C (Recovery)*

\[ y_t = \theta_{2,1}x_{1,t-m} + \theta_{2,2}x_{2,t-m} + \theta_{2,3}x_{3,t-m} \quad \text{if} \quad \psi_t \in C_2 \]  

(28)

where \( \theta_{i,j} \) represent parameters in mode \( i \) \((i = 0, 1, 2)\), and \( C_0, C_1 \) and \( C_2 \) represent subspaces in the space of regressor vector. In this modeling, \( \theta_{i,j} \) \((i = 0, 1, 2, j = 1, 2, 3)\) are unknown parameters. Also, the boundaries between subspaces \( C_0, C_1 \) and \( C_2 \) are unknown. This point makes the identification problem much more difficult than the standard parameter identification for the simple linear model.

By introducing this LS-ARX model, it becomes possible to capture not only the motion aspect (represented by coefficients \( \theta_{i,j} \) \((i = 0, 1, 2, j = 1, 2, 3)\) but also the decision making aspect (represented by boundaries between \( C_0, C_1 \) and \( C_2 \)) of the human behavior. The time interval to be analyzed was set to between the beginning of the steering operation and the ending of it.

**Results of clustering and identified parameters** We have analyzed the driving data of two male examinees (20s years old). The driver A carried out the avoidance with only steering, whereas the driver B used lots of braking together with the steering. The driving data used for the analysis is shown in Figs.5 to 10. Each data shows the profiles of three trials in each experiment set. The horizontal axis represents the range between cars, the vertical axes of the top, middle and bottom figure represent the steering, lateral displacement between cars and range rate, respectively. The right turn of the steering takes
Fig. 5. Examinee A — Set 1

Fig. 6. Examinee A — Set 2
Fig. 7. Examinee A — Set 3

Fig. 8. Examinee A — Set 4
Fig. 9. Examinee B — Set 1

Fig. 10. Examinee B — Set 2
positive value. The lateral displacement between cars takes positive value when the driver’s vehicle moves to right side against the preceding vehicle. The range rate is the difference of the speed between the drivers vehicle and the preceding truck. All data are normalized before analysis. We can see that all data are appropriately decomposed into three modes, i.e. the ‘Avoidance’, ‘Overtake’ and ‘Recovery’ show up in reasonable fashion. The parameter identification results for each mode are shown in Table 1. From Table 1, we can figure out the following driving characteristics:

**Mode A (Avoiding)**

The sign of $\theta_{0,1}$ (gain for the range) is negative, the sign of $\theta_{0,2}$ (gain for the range rate) is positive, and the sign of $\theta_{0,3}$ (gain for the lateral displacement) is almost negative. This is the common characteristics between drivers. On the other hand, we can see some difference between drivers in the magnitude of the gains in the mode A. Although the magnitude of the gain $\theta_{0,3}$ tends to be large in the driver A, the magnitude of the gain $\theta_{0,2}$ tends to be large in the driver B. This implies that the driver A signifies the information on the lateral displacements, whereas the driver B signifies the range rate in the mode A. This difference may be caused by the difference in the experience of riding the DS.

**Mode B (Overtake)**

The magnitude of $\theta_{1,1}$ (gain for the range) tends to become large compared with mode A. This indicates the fact that the drivers pay more attention to the range in the mode B than in the mode A.

**Mode C (Recovery)**

The parameter set found in this mode shows large variances. This implies that no common control theoretic understanding exists. This is probably because the drivers cannot find any clear target during the mode C, i.e. in the recovery mode.

**Table 1.** Identification results of parameters

<table>
<thead>
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<th></th>
<th>A-1</th>
<th>A-2</th>
<th>A-3</th>
<th>A-4</th>
<th>B-1</th>
<th>B-2</th>
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<tr>
<td>$\theta_{2,2}$</td>
<td>-0.840</td>
<td>-0.720</td>
<td>0.523</td>
<td>0.562</td>
<td>-0.453</td>
<td>-0.349</td>
</tr>
<tr>
<td>$\theta_{2,3}$</td>
<td>0.100</td>
<td>0.028</td>
<td>-0.865</td>
<td>1.003</td>
<td>-1.360</td>
<td>-0.540</td>
</tr>
</tbody>
</table>
Identified switching plane The identification results of the switching planes from one mode to other mode is shown in Table 2. The plane was supposed to have the form

\[ ax_1 + bx_2 + cx_3 = h. \]  

(29)

One of the obtained plane is shown in Fig.11. Note that \( x_1, x_2 \) and \( x_3 \) are normalized. The driver can be considered to switch the control law when he crosses this plane.

Table 2. Parameter on Hyperplane

<table>
<thead>
<tr>
<th></th>
<th>( A )</th>
<th>( B )</th>
<th>( C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>( b )</td>
<td>-1.28</td>
<td>-1.00</td>
<td>-1.42</td>
</tr>
<tr>
<td>( c )</td>
<td>-1.30</td>
<td>-0.83</td>
<td>-0.45</td>
</tr>
<tr>
<td>( h )</td>
<td>-0.50</td>
<td>-0.12</td>
<td>-0.72</td>
</tr>
</tbody>
</table>

Both drivers show the opposite sign in the coefficients \( a \) and \( b \) in the switching plane from the mode A (avoidance) to the mode B (overtake). This implies that the larger range rate leads to the earlier switching. Also, we can see the similar tendency in the relationship of coefficients \( a \) and \( c \). However, it is found that there is the difference in the magnitude of coefficient \( c \). The driver B shows the smaller magnitude of \( c \) than that of the driver A. This indicates that the driver B feels less significance in the lateral displacement in the case of switching from the mode A to the mode B. In the switching plane from the mode B to the mode C, both driver shows the large magnitude in the coefficients \( a \) and \( b \), and the small magnitude in the coefficient \( c \). This indicates that the switching from the mode B to the mode C is decided by referring the range and range rate.

The switching planes projected on \( x_1 - x_3 \) are shown in Fig.12 together with the behavior of the vehicles.

4.4 Identification as SS-ARX model

In this subsection, the SS-ARX model (three states, left-to-right model) is introduced as a mathematical model of the driving behavior. Another three drivers carried out the same experiment under virtual environments described in the previous section. The profiles of the observed steering and the estimated steering calculated from the obtained parameters of the corresponding SS-ARX model are depicted in Fig.13. The obtained parameters are listed in Table 3. Note
Fig. 11. Switching hyperplane

Fig. 12. Mode switching plane with vehicle behavior
that the two vertical lines in Fig.13 represent the most likely switching points calculated by using the following Viterbi algorithm (Recall that the number of discrete states is three).

\[
\delta(i, t) = \max_{0 \leq j \leq N} \{ \delta(j, t-1)a_{ji}b_i(o_t) \}
\]

\[
\gamma(i, t) = \arg\max_{0 \leq j \leq N} \{ \delta(j, t-1)a_{ji}b_i(o_t) \}
\]

where \(\gamma(i, t)\) represents the most likely state \(S_i\) at time \(t\). The estimated steering profile in Fig.13 is generated by regarding the SS-ARX model as a deterministic switched ARX model specified by \(\theta_i\) and switching points (given by \(\gamma(i, t)\)). From Fig.13, we can see that the parameter estimation algorithm works well. The identified parameters show similar tendency to the results obtained by applying the LS-ARX model.

**Drivers behavior recognition** Now, the obtained SS-ARX model for each driver is used as the ‘behavior recognizer’. The list of Logarithm of the likelihood value of each SS-ARX model for all observed data are listed in Table 4. In Table 4, SS-ARX(\(\ast\)) denotes the SS-ARX model obtained from the corresponding driver’s data profile. Also, \(O_\ast\) denotes the observed symbol sequence of the corresponding driver. As shown in this table, the likelihood values for other drivers data are smaller than that for corresponding drivers data (designated by underlines). These results clearly show the potential ability of the SS-ARX model to be used as the behavior recognizer of human being.

5 Conclusions

In this paper, we have developed the modeling strategy of the human driving behavior based on the Hybrid System model focusing on the driver’s collision avoidance maneuver. In our modeling, the control scenario of the human driver, that is, the mapping from the driver’s sensory information to the operation of the driver such as acceleration, braking and steering, was expressed by Piecewise ARX model. We have considered the cases that the switching between sub-ARX models is caused in both deterministic logic-based and stochastic manners. Then, the identification problems for the logic-based/stochastic switched ARX models (LS-ARX and SS-ARX models) were formulated and solved. Our experiments have clearly shown that the LS-ARX model enables us to capture not only the physical meaning of the driving skill, but also the decision-making aspect (switching conditions) in the driver’s collision avoidance maneuver. This can be an important guideline for the design of the driving assist system. Our experiments also have shown that the SS-ARX model can be used for the behavior recognition which can also be exploited to understand the humans intention. Although there exist some trial and error in the decision of the initial parameters in the EM algorithm, the data clustering idea developed for the LS-ARX model can be used for this purpose. Finally, the open problems are listed below:
Fig. 13. Steering profiles of three drivers (observed and estimated)
### Table 3. Identified parameters

#### SS-ARX(A)

<table>
<thead>
<tr>
<th>Parameters in ARX model</th>
<th>Variances</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_0 = (-0.824773, 2.244757, 0.603667)$</td>
<td>$\sigma_0 = 0.037493$</td>
</tr>
<tr>
<td>$\theta_1 = (-3.483122, 4.816041, -5.470960)$</td>
<td>$\sigma_1 = 0.011330$</td>
</tr>
<tr>
<td>$\theta_2 = (-3.936077, 4.499318, -4.582369)$</td>
<td>$\sigma_2 = 0.024018$</td>
</tr>
</tbody>
</table>

**A matrix**

$$a_{ij} = \begin{bmatrix} 0.999716 & 0.000284 & 0.000000 \\ 0.000000 & 0.977819 & 0.022181 \\ 0.000000 & 0.000000 & 1.000000 \end{bmatrix}$$

#### SS-ARX(B)

<table>
<thead>
<tr>
<th>Parameters in ARX model</th>
<th>Variances</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_0 = (-1.581261, 1.887177, -4.802167)$</td>
<td>$\sigma_0 = 0.032215$</td>
</tr>
<tr>
<td>$\theta_1 = (1.878792, -1.140327, 0.043192)$</td>
<td>$\sigma_1 = 0.008611$</td>
</tr>
<tr>
<td>$\theta_2 = (-1.100865, 0.516427, -1.362143)$</td>
<td>$\sigma_2 = 0.018346$</td>
</tr>
</tbody>
</table>

**A matrix**

$$a_{ij} = \begin{bmatrix} 0.994427 & 0.005573 & 0.000000 \\ 0.000000 & 0.998804 & 0.001196 \\ 0.000000 & 0.000000 & 1.000000 \end{bmatrix}$$

#### SS-ARX(C)

<table>
<thead>
<tr>
<th>Parameters in ARX model</th>
<th>Variances</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_0 = (-2.523657, 2.362553, -7.386183)$</td>
<td>$\sigma_0 = 0.149673$</td>
</tr>
<tr>
<td>$\theta_1 = (0.609914, -1.271813, 1.165948)$</td>
<td>$\sigma_1 = 0.004568$</td>
</tr>
<tr>
<td>$\theta_2 = (-0.066241, 0.187735, -0.645891)$</td>
<td>$\sigma_2 = 0.007644$</td>
</tr>
</tbody>
</table>

**A matrix**

$$a_{ij} = \begin{bmatrix} 0.999743 & 0.000257 & 0.000000 \\ 0.000000 & 0.992420 & 0.007580 \\ 0.000000 & 0.000000 & 1.000000 \end{bmatrix}$$

### Table 4. Recognition results

<table>
<thead>
<tr>
<th>SS-ARX model</th>
<th>$O_A$</th>
<th>$O_B$</th>
<th>$O_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS-ARX(A)</td>
<td>-731.673661</td>
<td>-4600.403127</td>
<td>-4910.527209</td>
</tr>
<tr>
<td>SS-ARX(B)</td>
<td>-2887.603213</td>
<td>-726.257733</td>
<td>-3711.502708</td>
</tr>
<tr>
<td>SS-ARX(C)</td>
<td>-3397.165848</td>
<td>-2780.210061</td>
<td>-725.213853</td>
</tr>
</tbody>
</table>
The mathematical structure of each submodel may be different in the human behavior. Especially, it is known that the human appropriately switches the ‘Feedforward control’ and ‘Feedback control’. This mechanism must be included in the system model.

The selection mechanism of the sensory information \(x_i\) must be developed. In this work, the sensory information were specified in advance. In more complicated situation, it is highly recommended to capture them automatically.

Some higher level model such as language model must be introduced to represent more complicated behavior.

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