Modeling of Human Driving Behavior based on Expression as Hybrid Dynamical System†


This paper develops a new framework to understand the human driving behavior based on the expression as a Hybrid Dynamical System (HDS) focusing on the driver’s stopping maneuver. The driving data are collected by using the three-dimensional driving simulator based on CAVE, which provides three-dimensional visual information. In our modeling, the relationship between the measured information such as distance to the stop line, its first and second derivatives and the braking amount are expressed by the Piecewise Polynomial System (PWPS) model, which is a class of HDS. The key idea to solve the identification problem is to formulate the problem as the Mixed Integer Linear Programming (MILP) with replacing the switching conditions by binary variables. From the obtained results, it is found that the driver appropriately switches the ‘control law’ according to the distance to the stop line. Our proposed approach enables us to capture not only the physical meaning of the driving skill, but also the decision-making aspect (switching conditions) in the driving behavior.

Key Words: CAVE, Virtual Reality, Three-dimensional Visual Information, PWPS, MILP, Driving Skill

1. Introduction

Recently, the modeling of the driving behavior has attracted a great deal of attention by many researchers1),2),3). Since the driving behavior usually includes high non-linearity, some nonlinear dynamics based modeling is promising. From this viewpoint, the nonlinear regression models, the neural network and the fuzzy system etc. can be used. If we use these techniques, however, the obtained model often results in too complicated model, and this makes it impossible to understand the physical meaning of the driver’s behavior. When we look at the driving behavior, it is often found that the driver appropriately switches the simple control laws instead of adopting complex nonlinear control law. This switching mechanism can be regarded as a kind of driver’s decision making in the driver’s behavior.

Therefore, it is highly recommended that the model of the driving behavior involve both physical skill (motion) and the decision-making aspect (switching condition). This kind of expression can be categorized into a class of Hybrid Dynamical Systems (HDSs). HDSs are systems, which consist of both continuous dynamics and logical conditions. The former are typically associated with the differential (or difference) equations, the latter with combinatorial logics, automata and so on. Although many literatures have dealt with the description, stability analysis, control and verification of the HDS, the identification problem for the HDS, that is finding the HDS model to represent the measured data, has not been fully discussed in the community of the HDS4),5),6),7).

In this paper, the Piecewise Polynomial Systems (PWPS) model, which is a class of the HDS, is adopted to understand the human driving behavior especially focusing on the stopping maneuver. The driving data are collected by using the three-dimensional driving simulator2),13) based on CAVE, which provides three-dimensional visual information. The major difference between our simulator and the real environment is the existence of the real motion of the vehicle. This implies that the acceleration factor may affect on the driving maneuver in different way. However, thanks to the 3D effect of the CAVE, this difference is not so large, i.e. the examinee shows the similar behavior in the stopping maneuver with one in the real environment7).

In our modeling, the relationship between the measured
information such as distance to the stop line, its first and second derivatives and the braking amount are expressed by the PWPS model. Then, we formulate the identification problem for the PWPS model as the Mixed Integer Linear Programming (MILP) by transforming the logical conditions into inequalities. By applying the developed strategy, it becomes possible to find not only coefficients in the polynomials but also parameters in the logical conditions from the measured driving data. This implies that both physical skill and the decision-making aspect in the driving behavior can be identified simultaneously.

This paper is organized as follows.

In Section 2, configuration of the developed DS based on the CAVE is introduced. In Section 3, the scenario of our examination is described. Based on the setup described in Section 3, driver’s stopping maneuver is investigated in Section 4. In Section 5, the modeling of the stopping maneuver based on the expression as the PWPS model is introduced, and identification results based on the MILP are shown.

2. Configuration of Driving Simulator

![Diagram of the Developed Driving Simulator](image)

Fig. 1 The developed driving simulator. (a) Configuration of the DS. (b) CAVE system.

The configuration and appearance of the developed DS based on the CAVE are shown in Fig. 1(a) and (b). The display unit in the CAVE system provides the 3D virtual environment, and it is controlled by ONYX2.

The display program was developed by making use of the CAVE library and the Performer. The cockpit is built by installing a handle, an accelerator and a brake in the CAVE system.

The information on the driver’s output to the handle, accelerator and brake is transferred to the PC through the USB terminal, and the vehicle position and motion are calculated based on these inputs and vehicle dynamics implemented on the PC using the CarSim software. The results of the calculation are transferred to ONYX2 through the Internet (TCP/IP), and the 3D visual image based on the position and motion of the vehicle is displayed.

3. Capturing Driving Behavior

3.1 Road environment and investigated task

Generally speaking, most traffic accidents occur at intersections\(^8\),\(^9\). In this paper, we focus on a stopping maneuver in front of the stop line because this maneuver strongly depends on the distance to the stop line, and our DS has clear advantage over other 2D virtual environment based DS\(^2\).

In order to model the stopping maneuver, the following sensory information is captured as the inputs:

(i) Distance from the vehicle to the stop line \([x_{1,k}]\)
(ii) First time derivative of (i) (velocity) \([x_{2,k}]\)
(iii) Second time derivative of (i) (acceleration) \([x_{3,k}]\)

The outputs of the driver are also specified as follows:

(i) Acceleration \([y_{1,k}]\)
(ii) Braking \([y_{2,k}]\)

Note that no steering operation is necessary in the stopping maneuver.

Since it is reported that the velocity \(x_{2,k}\) does not play an important role in the stopping maneuver\(^2\),\(^10\),\(^11\), (i) and (iii) of the sensory information and (ii) of the output of the driver are used for the modeling.

The configuration of the intersection and its projected image are shown in Fig. 2(a) and (b) with some geometric parameters.

The road is 7m wide and the pedestrian way is 2m wide. There are two intersections in this environment (A and B). The driver is supposed to stop in front of the stop line at the intersection (A). The vehicle is supposed to start moving at position (a or b).
3.2 Driving conditions

The vehicle used in the simulator is a large-size passenger car whose engine displacement is 3000cc. Eight male drivers who have driver’s licenses, ranging in age from 22 to 25 years, are selected as examinees. All examinees did not know the purpose of this experiment and we did not allow them to have any knowledge regarding this study. The maximum velocity is set to be 50km/h, and the selected starting point is 0m or 100m as shown in Fig. 2(a).

<table>
<thead>
<tr>
<th>Notation</th>
<th>Driving conditions(Maximum steady running velocity[km/h], start point[m])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preliminary drive</td>
<td>Ten times 50km/h 0m 50km/h 100m</td>
</tr>
<tr>
<td>Test drive</td>
<td>Three times Three times</td>
</tr>
</tbody>
</table>

Each driver takes a number of preliminary trials to get used to the DS, and then begins the test, which is made up of numerous trials. The number of trials for each driver is listed in Table 1.

3.3 Experimental procedures

Both preliminary trials and test trials are carried out twenty times and six times, respectively. These trials consist of two different driving situations.

1. Velocity restriction of 50km/h, starting point of 0m.
2. Velocity restriction of 50km/h, starting point of 100m.

The experiments are executed in the sequence shown in Table 1. After the preliminary trials, the driver rests for 10 minutes, and answers a questionnaire in which he record his impressions of his driving behavior, driving preferences, SSSQ, and so on. The SSSQ schedule is a way to find a driver who is likely to suffer from simulator sickness. If the driver wants to quit the experiment due to the simulator sickness, the experiment is suspended. Each driver takes about 30 minutes to complete all of the trials.

4. Experimental Results and Proposed Modeling Framework

Based on the setup described in section 3, eight drivers carried out the experiment under virtual environments.

Fig. 2 Approaching an intersection. (a) Model of intersection. (b) Sample of projected image.

Fig. 3 Behavior at stopping.

Firstly, the stopping maneuver, which is characterized by the profile between the beginning of the deceleration and the stopping point, were measured and analyzed. The profiles of the driving data of the six drivers in the case that the velocity restriction is 50km/h are depicted in Fig. 3 (the data of third and fourth trials with 50km/h,
The horizontal and vertical axes in Fig. 3 denote the position (stop line is located at 300m) and the velocity of the vehicle, respectively. When we look at profiles in Fig. 3, roughly speaking, the stopping pattern can be regarded as the series of the following three sub-maneuvers: (1) deceleration, (2) running with constant velocity and (3) re-deceleration. This sequence of sub-maneuvers is often observed in the driving pattern in real cars. In Fig. 3(a), however, the drivers E2 and E3 show different maneuver from one described above. From the answer to the questionnaire, we could find the reason of this as follows: The driver (E2) often makes ‘sudden stopping’ in his real driving situation. Also, the driver (E3) has found the optimal beginning point for the deceleration by investigating the building and the electric pole around the stop line instead of taking a look at the stop line in this experiment.

In the following, we call the duration corresponding to above three sub-maneuvers ‘Interval A’, ‘Interval B’ and ‘Interval C’.

The more formal expression based on the polynomial model is introduced in the following.

**The interval A (deceleration)**

\[ y_{2,k} = a_0x_{1,k} + a_1x_{3,k} + a_2x_{1,k}^2 + a_3x_{3,k}^2 \]  \hspace{1cm} (1)

if \( d_1 > x_{1,k} \)

**The interval B (running with constant velocity)**

\[ y_{2,k} = b_0x_{1,k} + b_1x_{3,k} + b_2x_{1,k}^2 + b_3x_{3,k} \]  \hspace{1cm} (2)

if \( d_2 > x_{1,k} \geq d_1 \)

**The interval C (re-deceleration)**

\[ y_{2,k} = c_0x_{1,k} + c_1x_{3,k} + c_2x_{1,k}^2 + c_3x_{3,k} \]  \hspace{1cm} (3)

if \( x_{1,k} \geq d_2 \)

In eqs. (1) to (3), \( x_{1,k} \) and \( x_{3,k} \) denote the position and acceleration, respectively. \( d_1 \) and \( d_2 \) are parameters to specify the switching between intervals. Note that only \( x_{1,k} \) and \( x_{3,k} \) appear in the left-hand side in eqs. (1) to (3). In our past study \(^2\), we have clarified that \( x_{1,k} \) and \( x_{3,k} \) play important roles in the stopping maneuver by applying the GMDH technique. Also, the number of switching may be reduced to explain the behavior found in examinees E2 and E3. This behavior, however, can be explained naturally by setting the distance between \( d_1 \) and \( d_2 \) to be small. Since the structure of this model contains both the continuous dynamics (polynomials) and the logical conditions (switching of polynomials), the proposed model belongs to a kind of Hybrid Dynamical Systems (HDS).

As alternative ways of modeling the driving maneuver, for example, nonlinear regression models, neural network and fuzzy system can be used. If we use these techniques, however, the obtained model often results in too complicated model, and this makes it impossible to understand the physical meaning of the driver’s behavior. On the contrary, the HDS model proposed in this section enables us to capture not only the physical meaning (polynomials), but also the decision-making aspect (logical conditions) in the driving maneuver. Note that the switching conditions between each interval are not specified in advance in our model. The parameter specifying switching condition ( \( d_1 \) and \( d_2 \) ) and coefficients appearing in each polynomial ( \( a_i \) to \( c_i \) ) must be found simultaneously from the measured data. In the next section, the strategy to solve this simultaneous identification problem is developed.

5. Modeling of Driver’s Behavior Based on Piece-Wise Polynomial System (PWPS) Expression

5.1 Expression as PWPS

PWPS is one of classes of hybrid system, in which logic, dynamics and constraints are integrated. PWPS is formulated by (4).

\[
y_{2,k} = \begin{cases} 
  a_0x_{1,k} + a_1x_{3,k} + a_2x_{1,k}^2 + a_3x_{3,k}^2 & \text{if } d_1 > x_{1,k} \\
  b_0x_{1,k} + b_1x_{3,k} + b_2x_{1,k}^2 + b_3x_{3,k} & \text{if } d_2 > x_{1,k} \geq d_1 \\
  c_0x_{1,k} + c_1x_{3,k} + c_2x_{1,k}^2 + c_3x_{3,k} & \text{if } x_{1,k} \geq d_2 
\end{cases} 
\]  \hspace{1cm} (4)

where \( k \in Z \) is a discrete time. Thus, the PWPS form includes both continuous dynamics (polynomials) and logical conditions. The goal of our modeling is to find not only coefficients in the polynomials \( a_i \), \( b_i \) and \( c_i \) but also parameters in the “switching conditions” \( d_i \) from the measured driving data. Although this identification problem is not straightforward to handle, the idea developed in the
Mixed Logical Dynamical Systems framework [4] makes it tractable. The key idea is to transform the logical condition into some inequalities by introducing auxiliary binary variables \( \delta \in \{0, 1\} \) and auxiliary continuous variables \( z \), and to formulate the problem as the Mixed Integer Linear Programming.

5.2 Useful tools for transformation from logical condition into inequalities

In this section, some useful tools to transform the logical condition into inequalities are introduced. Firstly, the logical relationship given by

\[ f(x) \geq a \iff [\delta = 1] \]  

(5)

can be transformed into inequalities (6) and (7).

\[ -f(x) + (a - M)\delta + m \leq 0 \]  

(6)

\[ f(x) - (M - a + \epsilon)\delta - a + \epsilon \leq 0 \]  

(7)

where \( M = \max_x f(x) \), \( m = \min_x f(x) \), and \( \epsilon > 0 \) is a small tolerance. Also, in our setup, the product term of binary and continuous variables such as \( \delta f(x) \) often appear. Since it is undesirable to handle this nonlinear term, we secondly introduce another auxiliary variable \( z = \delta f(x) \), which satisfies the following two logical relationships.

\[ [\delta = 0] \Rightarrow [z = 0], [\delta = 1] \Rightarrow [z = f(x)] \]  

(8)

These relationships can be transformed into the following equivalent inequalities.

\[ z \leq M \delta \]  

(9)

\[ -z \leq -M \delta \]  

(10)

\[ z \leq f(x) - m(1 - \delta) \]  

(11)

\[ -z \leq -f(x) + M(1 - \delta) \]  

(12)

5.3 Identification of PWPS model by Mixed Integer Linear Programming (MILP)

In order to transform the three logical conditions involved in (4) into the equivalent inequalities, binary variables \( \delta_1, \delta_2 \) are introduced as follows:

(1) \( [d_1 > x_{1,k}] \iff [\delta_{1,k} = 0, \delta_{2,k} = 0] \)

(2) \( [d_2 > x_{1,k}] \iff [\delta_{1,k} = 1, \delta_{2,k} = 0] \)

(3) \( [x_{1,k} > d_2] \iff [\delta_{1,k} = 0, \delta_{2,k} = 1] \)

By applying the transformation rules and introducing the auxiliary variables, eq. (4) can be reformulated by the following linear equation.

\[
y_{2,k} = a_0 x_{1,k} + a_1 x_{3,k} + a_2 x_{4,k}^2 + a_3 x_{5,k}^3 + x_{1,k} z_{1,k} + x_{3,k} z_{2,k} + x_{4,k}^2 z_{3,k} + x_{5,k}^3 z_{4,k} + x_{1,k}^2 z_{5,k} + x_{3,k} z_{6,k} + x_{4,k}^2 z_{7,k} + x_{5,k}^3 z_{8,k} \tag{13}
\]

where, the auxiliary variables, \( z_{i,k} \), are defined as follows.

\[
z_{i,k} = \delta f_{i,k} = \delta_j(b_i - a_i) \]

(14)

\[
z_{i,k} = \delta f_{i,k} = \delta_j(c_i - a_i) \]

(15)

\[
z_{i,k} = \delta f_{i,k} = \delta_j(d_i) \]

(16)

\[
z_{i,k} = \delta f_{i,k} = \delta_j d_i \]

(17)

\[
z_{i,k} = \delta f_{i,k} = \delta_j d_i \]

Now, the problem to find the parameters in the switching condition and coefficients in the polynomials is formulated as the following MILP.

\[
\text{known } y_{2,k}, x_{1,k}, x_{3,k} \\
\text{find } \{a_0, a_1, a_2, a_3, b_0, b_1, b_2, b_3, c_0, c_1, c_2, d_1, d_2, \delta_{1,k}, \delta_{2,k}\} \\
\text{which minimize } J = \sum_{k=1}^{N} |y_{2,k} - \hat{y}_{2,k}| \tag{18}
\]

subject to

\[
z_{i,k} \leq M_i \delta_{j,k} \]  

(19)

\[
-z_{i,k} \leq -m_i \delta_{j,k} \]  

(20)

\[
z_{i,k} \leq f_{i,k} - (1 - \delta_{j,k})m_i \]  

(21)

\[
-z_{i,k} \leq -f_{i,k} + (1 - \delta_{j,k})M_i \]  

(22)

\[
0 \leq \delta_{1,k} \leq 1 \]  

(23)

\[
0 \leq \delta_{2,k} \leq 1 \]  

(24)

\[
0 \leq \delta_{1,k} + \delta_{2,k} \leq 1 \]  

(25)

\[
\delta_{2,k} \leq \delta_{2,k+1} \]  

(26)

\[
x_{1,k} \leq d_1 - z_{9,k} + z_{10,k} - z_{11,k} - \epsilon + \delta_{1,k}(M_d + \epsilon) \]  

(27)

\[
x_{1,k} \geq (1 - (\delta_{1,k} + \delta_{2,k})) m_d + z_{9,k} + z_{12,k} \]  

(28)

where, \( M_i \) and \( m_i (i = 1 \sim 12) \) are the maximum and minimum values of \( z_{i,k} \), and \( M_d \) and \( m_d \) represent the maximum and minimum values of \( d_1 \) and \( d_2 \), respectively, \( \epsilon \) is a small tolerance.

There are several ways to solve the MILP. One of the most efficient algorithms is a branch-and-bound method. Although it requires some heuristic rules in the decision of the branching variable, it can guarantee the optimality, and can reduce the computational burden with assistance of appropriate heuristic rules.

Note that the computational burden strongly depends on the number of binary variables since this specifies the size of the search space. In our case, the number of the measurement point affects the computational burden significantly.
5.4 Identification results of stopping behavior based on PWPS model

Based on the formulation of the identification of the PWPS model described in the previous section, the identification of parameters in the switching conditions and coefficients in the polynomials in the stopping behavior is carried out. The profile between the beginning of the deceleration and the final stopping point of each driver shown in Fig. 3 was used for the modeling.

![Fig. 3](image)

Fig. 3 Comparison of actual brake input and estimated brake input.

The fifteen sampling data of the third trial of six drivers were used for the identification of the stopping behavior. These data were selected by culling from the measured data between the beginning of the deceleration and the final stopping point. Before applying the MILP to the measured data, the input and output data were normalized as follows:

\[
\bar{x}_i = \begin{cases} 
\frac{x_i}{x_{\text{max}}} & (x_i \geq 0) \\
\frac{x_i}{x_{\text{min}}} & (x_i < 0)
\end{cases}, \quad y_i = \begin{cases} 
\frac{y_i}{y_{\text{max}}} & (y_i \geq 0) \\
\frac{y_i}{y_{\text{min}}} & (y_i < 0)
\end{cases}
\]

(29) (30)

All numerical experiments have been performed by PC (CPU 4 3.06[GHz] and Memory 1024[MB]). It took about two hours to find the solution of the MILP. In order to verify the validity of the obtained PWPS model, the reproduced braking amount, which is calculated using the MILP, and the measured braking amount and the velocity of the vehicle are plotted in Fig. 4.

In Fig. 4, the horizontal axis represents the distance from the starting point. The left and right side vertical axes represent the velocity and braking amount of the vehicle, respectively. The stopping line is located at 300[m]. Note that the braking amount takes negative values, and zero braking implies ‘no braking’. Also, the switching points between polynomials are designated by vertical lines. As shown in Fig. 4, the measured braking amount and reproduced braking amount based on the PWPS model agree well with each other. These results verify the effectiveness of the modeling based on the PWPS. It is interesting that the switching points, however, vary from driver to driver. In order to understand the common characteristics in the stopping behavior, the identified parameters for three examinees (E2, E4 and E6) by solving the MILP are listed in Table 3.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values(E2)</th>
<th>Values(E4)</th>
<th>Values(E6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a0</td>
<td>-0.4619</td>
<td>-0.1501</td>
<td>0.9824</td>
</tr>
<tr>
<td>a1</td>
<td>2.3118</td>
<td>1.1358</td>
<td>1.2902</td>
</tr>
<tr>
<td>a2</td>
<td>0.5474</td>
<td>0.1834</td>
<td>-1.1310</td>
</tr>
<tr>
<td>a3</td>
<td>4.4584</td>
<td>0.8705</td>
<td>1.8742</td>
</tr>
<tr>
<td>b0</td>
<td>26.8100</td>
<td>24.2000</td>
<td>1.7534</td>
</tr>
<tr>
<td>b1</td>
<td>1.6844</td>
<td>-20.6600</td>
<td>0.4065</td>
</tr>
<tr>
<td>b2</td>
<td>-27.5780</td>
<td>-34.4040</td>
<td>-1.8845</td>
</tr>
<tr>
<td>b3</td>
<td>-1.2212</td>
<td>-12.058</td>
<td>-0.6546</td>
</tr>
<tr>
<td>c0</td>
<td>-0.0241</td>
<td>-2.0050</td>
<td>11.2200</td>
</tr>
<tr>
<td>c1</td>
<td>-0.0233</td>
<td>0.0039</td>
<td>-20.6630</td>
</tr>
<tr>
<td>c2</td>
<td>1.0139</td>
<td>1.0067</td>
<td>-20.8540</td>
</tr>
<tr>
<td>c3</td>
<td>-0.0122</td>
<td>0.0023</td>
<td>-10.029</td>
</tr>
<tr>
<td>d1</td>
<td>290.900</td>
<td>291.360</td>
<td>274.739</td>
</tr>
<tr>
<td>d2</td>
<td>299.477</td>
<td>296.776</td>
<td>290.222</td>
</tr>
</tbody>
</table>

The coefficients in the polynomials \(a_i, b_i, c_i\) and the parameters in the switching conditions \(d_i\) in (13) and (18) are
listed in Table 3. As shown in Table 3, the most dominant input information (designated by underline in Table 3) in the intervals B and C were the distance $x_{1,k}$ to the stop line, and the acceleration $x_{3,k}$ did not play an important role in the stopping maneuver. On the other hand, the acceleration $x_{3,k}$ was found to be dominant input information, and played a crucial role in the interval A. Similar tendency has been found in the identified results of other drivers.

From these observation, we can conclude that the driver appropriately switches the ‘control law’ according to the following scenario: At the beginning of the stopping maneuver (just after finding the stopping point), the driver decelerate the vehicle based on the acceleration information, and then switch to another control law based on the distance to the stop line. Although the switching points depend on the driver’s characteristics, qualitatively speaking, the scenario described above can be found as common characteristics in all drivers.

These results highly demonstrate the usefulness of the modeling based on the expression as an HDS.

6. Conclusions

A new framework to understand the human driving behavior based on the expression as HDS focusing on the driver’s stopping maneuver has been developed. The driving data have been collected by using the three-dimensional driving simulator based on CAVE, which provides three-dimensional visual information. In our modeling, the relationship between the measured information such as distance to the stop line, its first and second derivatives and the braking amount has been expressed by the PWPS model, which is a class of HDS. The key idea to solve the identification problem was to formulate the problem as the MILP with replacing the switching conditions by binary variables. From the obtained results, it was found that the driver appropriately switches the ‘control law’ according to the following scenario: At the beginning of the stopping behavior (just after finding the stopping point), the driver decelerate the vehicle based on the acceleration information, and then switches to the control law based on the distance to the stop line. Thus, our proposed approach enables us to capture not only the physical meaning of the driving skill, but also the decision-making aspect (switching conditions) in the driving behavior.

The analysis in more complicated situation, and the application of the obtained results to the design of Driving Assisting System are our future works.

Acknowledgement

This work was supported by the Space Robotic Center of the Toyota Technological Institute, where CAVE is installed. The authors would like to thank Dr. Yoji Umetani and the researchers involved at the Space Robotic Center for their helpful suggestions. The authors also would like to thank Dr. Yasushi Amano, of Toyota Central Research and Development Labs, for his valuable advices.

References

計測 太郎 (Member)

19xx年××大学××学部××工学科卒業、同年○○大学××工学科助手、現在に至る。計測理論、自動制御の研究に従事（工学博士）。

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